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Application of Generalized Exponential Function Method for Exact Solutions of Wu-Zhang System

Sajjad A. Jedi Abduridha [1], Mohammad Mirzazadeh [2], Samira Heidari [3], Mostafa Eslami [3]*

[1] Department of Mathematics, University of Kufa, Najaf, Iraq.

[2] Department of Engineering Sciences, University of Guilan, Rudsar, Iran.

[3] Department of Mathematics, University of Mazandaran, Babolsar, Iran.

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Abstract:
The present research aims to find a precise method for the wu-zhang system on scattered long waves, which can be a positive step for physical science in dealing with the structure of scattered waves and provide 3D diagrams for further studies of other sciences.

***Corresponding Author:** mostafa.eslami@umz.ac.ir

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1. Introduction

One of the most popular, widely used, and diverse mathematical equations is differential equations with partial derivatives, which sometimes attract the attention of many researchers and mathematical scientists who, by researching these equations and completing the research of others, arrive at remarkable methods and techniques that are a significant help to other sciences significantly in finding the answers to these equations.

Among the methods used for differential equations with partial derivatives that have been obtained over many years and as a result of the experiences of many scientists, Rasul Shah et al. 2019, used the Laplace-Adomin decomposition method to get the solutions of partial differential equations. They investigated the third-order diffuse fraction [1] and also applied the traveling wave transform of the non-differentiable type in 2019. Yang and Tenreiro Machado [2] discussed the exact solutions to the new nonlinear Burgers equation. In 2020, Touchent et al. applied a modified fixed subspace method to obtain the solution of partial differential equations with fractional derivatives of non-singular kernels [3], an attractive method for coupled systems of fractional partial differential equations which was proposed in 2020 by Al-Smadi et al.[4]. Guo et al. (2020) conducted research on deep learning and physical constraints for solving partial differential equations [5]. In 2021, three common methods of two M-fractional differential equations were adopted by Siddique et al., who applied and compared them [6]. A new neural network method for solving ordinary and partial differential equations was used in 2021 by Schiassi et al. [7], a generalized operational matrix to obtain the solutions of differential equations. The fractional fraction was investigated several times in 2022 by Imran Talib et al. [8]. In 2023, Ozkan and Ozkan presented a discussion of the exact solutions of a type of differential equation [9] and many pieces of research that have been carried out in previous years or later by many researchers.

Various forms of nonlinear partial differential equations have been studied for this purpose. For example, the nonlinear Schro¨ dinger equation [10], the Biswas- Milock equation [11], the Schro¨ dinger-Klein-Gordon (SKG) equation [12], the Dihrenfeld-Sokoloff equations [13], Tzitzika-type evolutionary equations [14], and many others.

This research investigates and studies one of these PDEs that appears in the dynamics of long scattered waves, known

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as the Wu Zhang model [15]. To solve this system, various powerful techniques and methods have been used:

- In 2002, Chen et al., with the help of Jacobi elliptic functions [16],
- In 2003, Zheng et al. and with the Extended tanh-function method [17],
- In 2011, Taghizadeh et al. and with the Reduced differential transform method [18],
- In 2015, Eslami et al., with the First integral method [19],
- In 2016, Inc et al. used two extended Tanh methods: the Hirota method [20]
- In 2016, Xiong et al., using the Lie symmetry method [21],
- In 2017, Mirzazadeh et al., by a method based on Lie symmetry [22],
- In 2017, Koparan et al. with the Generalized Kudryashov Method [23],
- In 2017, Kaplan et al., with the Exponential rational function method [24],
- In 2019, by Awan et al., using the generalized (G0=G) expansion method [25],
- In 2019, Yel and Baskonus, his colleague, modified the exp-expansion function method [26] and …

In the current research, we use the generalized exponential function method that Ghanbari and Inc. studied in 2018 [27] to reach new and accurate solutions for the wu-zhang system. This generalized exponential function method has already been used by Ghanbari and several researchers for many equations [28-34]. The outline of this work is as follows: In the second part, the key concepts of the generalized exponential function method are introduced. In the third part, we use this method for the above system. The fourth section includes results and discussion.

2. Demarche

The method used in the present research is a method that has been widely used in solving many PDE schemes, which we now use to obtain the solutions of the Wu-Zhang system model. To illustrate the technique used in this research, let us presume a framework as conform below:

$$H(X, X_x, X_t, X_{xx}, \cdots) = 0 \tag{1}$$

Using the wave $X(t,h) = X(\zeta)$, $\zeta = t - kh$, Equation 1 becomes a NODE equation

$$H\left(X(\zeta), \frac{dX}{dX}, \frac{d^2X}{dX^2}, \cdots\right) = 0 \tag{2}$$

This technique is built using a hypothetical solution that can be displayed as follows:

$$\mathcal{W}(\zeta) = \mathcal{V}_0 + \sum_{i=0}^{a_0} \mathcal{V}_i \mathbb{L}^i(\zeta) + \sum_{i=0}^{a_0} \frac{\mathcal{Z}_i}{\mathbb{L}^i(\zeta)} \tag{3}$$

where

$$\zeta = \frac{m_1 \exp(\gamma_1 \zeta) + m_2 \exp(\gamma_2 \zeta)}{m_3 \exp(\gamma_3 \zeta) + m_4 \exp(\gamma_4 \zeta)} \tag{4}$$

In Equation 4, $\mathcal{V}_0$, $\mathcal{V}_i$, $\mathcal{Z}_i$ $(1 \le i \le a_0)$, and in addition m_i, γ_i, $(1 \le i \le 4)$, to determine the positive integer a_0, we can use some well-known balance rules. By inserting Equation 3 in Equation 2, we get a polynomial. Finally, analytical solutions for Equation 1 are obtained.

3. Solution of Equation 1 with Demarche Suggested in Section 2

The Wu Zhang equation (WZ) studied in the present study is as follows:

$$\begin{cases} u_t + u_{xx} + \mathcal{W}u_y + X_x = 0 \\ \mathcal{W}_t + u\mathcal{W}_x + \mathcal{W}_y + X_y = 0 \\ X_t + (uX)_x + (\mathcal{W}X)_y + \frac{1}{3}(u_{xxx} + u_{xxy} + \mathcal{W}_{xxy} + \mathcal{W}_{yyy}) = 0 \end{cases} \tag{5}$$

By changing the scale and reducing symmetry, Equation 5 can be reduced to Equation 6:

$$\begin{cases} \mathcal{W}_t + \mathcal{W}\mathcal{W}_x + X_x = 0 \\ X_t + X\mathcal{W}_x + \mathcal{W}X_x + \frac{1}{3}\mathcal{W}_{xxx} = 0 \end{cases} \tag{6}$$

In this section, we want to find the exact solutions of the equation (WZ). For this purpose, from the traveling wave transformation $\mathcal{W}(t,h) = \mathcal{W}(\zeta)$, $X(t,h) = X(\zeta)$, $\zeta = t - kh$, system 6 is reduced to the following ODE system:

$$-c\mathcal{W}' + \mathcal{W}\mathcal{W}' + X' = 0 \tag{7}$$

$$-cX' + X\mathcal{W}' + \mathcal{W}X' + \frac{1}{3}\mathcal{W}''' = 0 \tag{8}$$

By integrating Equation 8 once and substituting X into Equation 9, we find:

$$(-3c^2 + 3\alpha)\mathcal{W}' + 9c\mathcal{W}\mathcal{W}' - \frac{9}{2}\mathcal{W}\mathcal{W}' + \mathcal{W}''' = 0 \tag{9}$$

$$\mathcal{W}'' - \beta - (3c^2 - 3\alpha)\mathcal{W} - \frac{9}{2}c\mathcal{W}^2 + \frac{3}{2}\mathcal{W}^3 = 0 \tag{10}$$

According to the balance principle, the value of a_0 is 1, so:

$$\mathcal{W}(\zeta) = \mathcal{V}_0 + \mathcal{V}_1\mathbb{L}(\zeta) + \frac{\mathcal{Z}_1}{\mathbb{L}(\zeta)} \tag{11}$$

That $\mathbb{L}(\zeta)$ is obtained according to Equation 4.

Clique 1: catching $[m_1 \quad m_2 \quad m_3 \quad m_4] = [-1 \quad 1 \quad 1 \quad 1]$ and $[\gamma_1 \quad \gamma_1 \quad \gamma_1 \quad \gamma_1] = [1 \quad -1 \quad 1 \quad -1]$ in Equation 4 render:

$$\mathbb{L}(\zeta) = -tanh\zeta \tag{12}$$

Various anthologies are obtained as follows:

Anthology 1-1:

$$\alpha = -\frac{c^2}{2} + \frac{2}{3}, \beta = -\frac{(3c^2) \times c}{2}, \mathcal{V}_0 = c, \\ \mathcal{V}_1 = \frac{2\sqrt{3}}{3}, \mathcal{Z}_1 = 0 \tag{13}$$

By substituting the values of the obtained parameters Equation 13 in Equation 11, the following wave solution is obtained:

$$\mathcal{W} = c - \frac{2\sqrt{3}\tanh(\zeta)}{3} \tag{14}$$

The graphical diagram related to the solution of the wave $\mathcal{W}(\zeta)$ while $c = 1$ is presented in Figure 1.

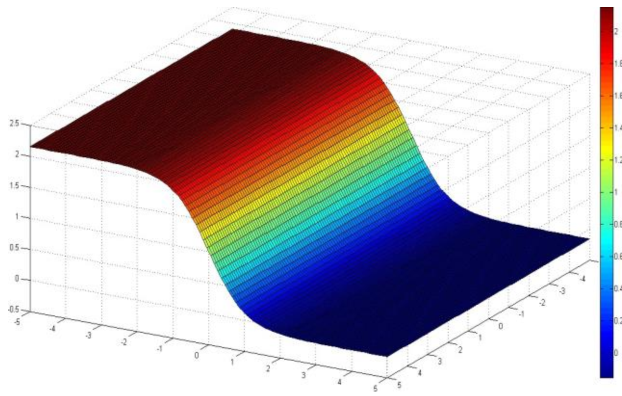

Figure 1. The graphical diagram related to the solution of Anthology 1-1 with c=1

Anthology 1-2:

$$\alpha = -\frac{c^2}{2} + \frac{8}{3}, \beta = -\frac{(3c^2+16)\times c}{2}, \mathcal{V}_0 = c,$$
$$\mathcal{V}_1 = \frac{2\sqrt{3}}{3}, \mathcal{Z}_1 = \frac{2\sqrt{3}}{3} \tag{15}$$

By substituting the values of the obtained parameters Equation 15 in Equation 11, the following wave solution is obtained:

$$\mathcal{W}(\zeta) = \frac{2\sqrt{3}\tanh(\zeta)^2 + 3c\tanh(\zeta) - 2\sqrt{3}}{3\tanh(\zeta)} \tag{16}$$

The graphical diagram related to the solution of the wave $\mathcal{W}(\zeta)$ while $c = 2$ is presented in Figure 2.

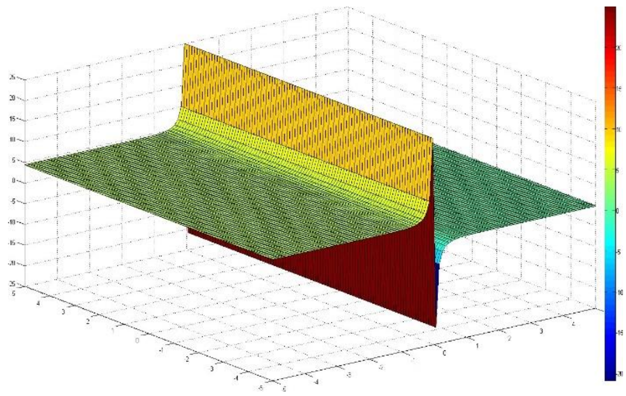

Figure 2. The graphical diagram related to the solution of Anthology 1-2 with c=2

Anthology 1-3:

$$\alpha = -\frac{c^2}{2} - \frac{4}{3}, \beta = -4c\frac{3}{2}c^3, \mathcal{V}_0 = c,$$
$$\mathcal{V}_1 = \frac{2\sqrt{3}}{3}, \mathcal{Z}_1 = -\frac{2\sqrt{3}}{3} \tag{17}$$

By substituting the values of the obtained parameters Equation 17 in Equation 11, the following wave solution is obtained:

$$\mathcal{W}(\zeta) = \frac{-2\sqrt{3}\tanh(\zeta)^2 + 3c\tanh(\zeta) + 2\sqrt{3}}{3\tanh(\zeta)} \tag{18}$$

The graphical diagram related to the solution of the wave $\mathcal{W}(\zeta)$ while $c = 2$ is presented in Figure 3.

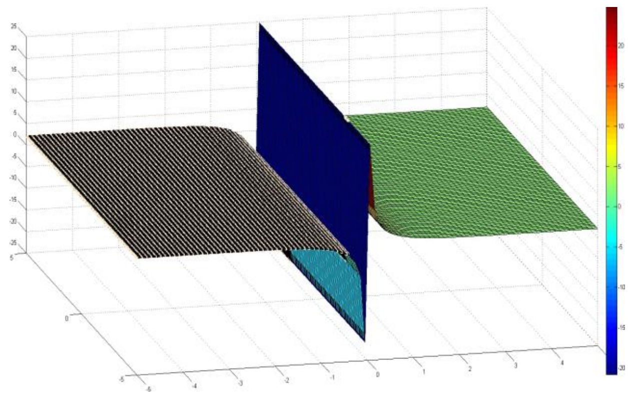

Figure 3. The graphical diagram related to the solution of Anthology 1-3 with c=2

Anthology 1-4:

$$\alpha = -\frac{c^2}{2} + \frac{2}{3}, \beta = -\frac{(3c^2-4)\times c}{2}, \mathcal{V}_0 = c,$$
$$\mathcal{V}_1 = 0, \mathcal{Z}_1 = -\frac{2\sqrt{3}}{3} \tag{19}$$

By substituting the values of the obtained parameters Equation 19 in Equation 11, the following wave solution is obtained:

$$\mathcal{W}(\zeta) = \frac{3\tanh(\zeta) - 2\sqrt{3}}{3\tanh(\zeta)} \tag{20}$$

The graphical diagram related to the solution of the wave $\mathcal{W}(\zeta)$ while $c = 3$ is presented in Figure 4.

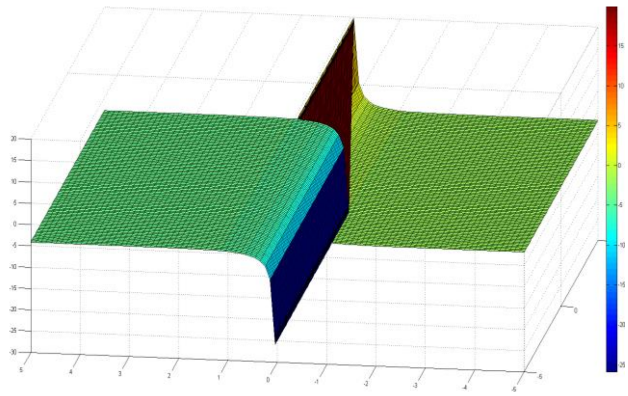

Figure 4. The graphical diagram related to the solution of Anthology 1-4 with c=3

Clique 2: catching $[m_1 \quad m_2 \quad m_3 \quad m_4] = [i \quad -i \quad 1 \quad 1]$ and $[\gamma_1 \quad \gamma_1 \quad \gamma_1 \quad \gamma_1] = [i \quad -i \quad i \quad -i]$ in Equation 4 render:

$$\mathbb{L}(\zeta) = -tanh\zeta \tag{21}$$

Anthology 2-1:

$$\alpha = -\frac{c^2}{2} - \frac{2}{3}, \beta = -\frac{(3c^2-4)\times c}{2}, \mathcal{V}_0 = c, \tag{22}$$

$\mathcal{V}_1 = -\frac{2\sqrt{3}}{3}, \mathcal{Z}_1 = 0$

By substituting the values of the obtained parameters Equation 22 in Equation 11, the following wave solution is obtained:

$$\mathcal{W} = c - \frac{2\sqrt{3}\tanh(\zeta)}{3} \tag{23}$$

The graphical diagram related to the solution of the wave $\mathcal{W}(\zeta)$ while $c = 4$ is presented in Figure 5.

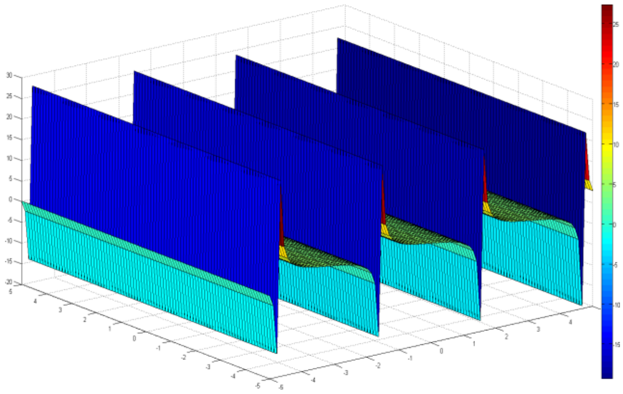

Figure 5. **The graphical diagram related to the solution of Anthology 2-1 with c=4**

Anthology 2-2:

$$\alpha = -\frac{c^2}{2} - \frac{8}{3}, \beta = -8c - \frac{3}{2}c^3, \mathcal{V}_0 = c,$$
$$\mathcal{V}_1 = -\frac{2\sqrt{3}}{3}, \mathcal{Z}_1 = \frac{2\sqrt{3}}{3} \tag{24}$$

By substituting the values of the obtained parameters Equation 24 in Equation 11, the following wave solution is obtained:

$$\mathcal{W}(\zeta) = \frac{2\sqrt{3}\tanh(\zeta)^2 + 3c\tanh(\zeta) - 2\sqrt{3}}{3\tanh(\zeta)} \tag{25}$$

The graphical diagram related to the solution of the wave $\mathcal{W}(\zeta)$ while $c = 5$ is presented in Figure 6.

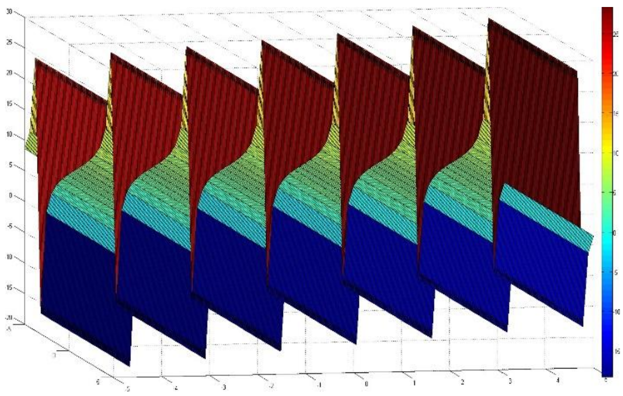

Figure 6. **The graphical diagram related to the solution of Anthology 2-2 with c=5**

Anthology 2-3:

$$\alpha = -\frac{c^2}{2} + \frac{4}{3}, \beta = 4c - \frac{3}{2}c^3, \mathcal{V}_0 = c, \tag{26}$$

$\mathcal{V}_1 = -\frac{2\sqrt{3}}{3}, \mathcal{Z}_1 = -\frac{2\sqrt{3}}{3}$

By substituting the values of the obtained parameters Equation 26 in Equation 11, the following wave solution is obtained:

$$\mathcal{W} = \frac{2\sqrt{3}\tanh(\zeta)^2 + 3c\tanh(\zeta) + 2\sqrt{3}}{3\tanh(\zeta)} \tag{27}$$

The graphical diagram related to the solution of the wave $\mathcal{W}(\zeta)$ while $c = 5$ is presented in Figure 7.

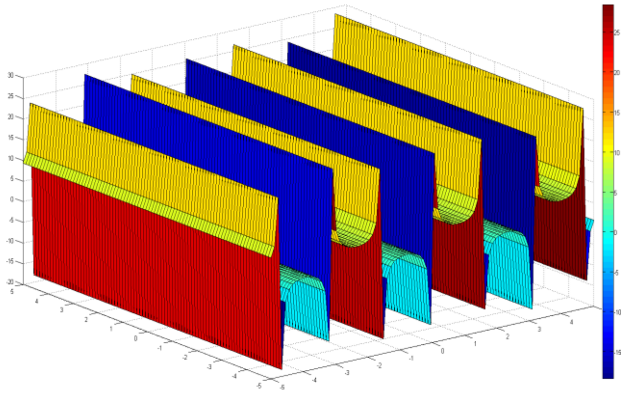

Figure 7. **The graphical diagram related to the solution of Anthology 2-3 with c=5**

Anthology 2-4:

$$\alpha = -\frac{c^2}{2} - \frac{2}{3}, \beta = -\frac{(3c^2+4)\times c}{2}, \mathcal{V}_0 = c,$$
$$\mathcal{V}_1 = 0, \mathcal{Z}_1 = -\frac{2\sqrt{3}}{3} \tag{28}$$

By substituting the values of the obtained parameters Equation 28 in Equation 11, the following wave solution is obtained:

$$\mathcal{W}(\zeta) = \frac{3\tanh(\zeta) + 2\sqrt{3}}{3\tanh(\zeta)} \tag{29}$$

The graphical diagram related to the solution of the wave $\mathcal{W}(\zeta)$ while $c = 3$ is presented in Figure 8.

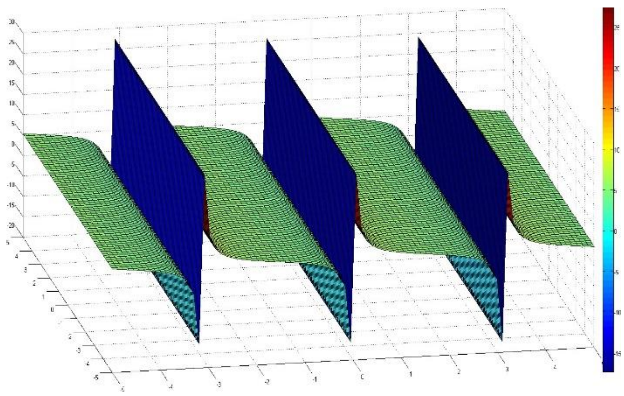

Figure 8. **The graphical diagram related to the solution of Anthology 2-4 with c=1**

Clique 3: catching $[m_1 \;\; m_2 \;\; m_3 \;\; m_4] = [1 \;\; 1 \;\; -1 \;\; 1]$ and $[\gamma_1 \;\; \gamma_1 \;\; \gamma_1 \;\; \gamma_1] = [1 \;\; -1 \;\; 1 \;\; -1]$ in Equation 4 render:

$$\mathbb{L}(\zeta) = -coth\zeta \tag{30}$$

Anthology 3-1:

$$\alpha = -\frac{c^2}{2} + \frac{2}{3}, \beta = -\frac{(3c^2-4)\times c}{2}, \mathcal{V}_0 = c,$$
$$\mathcal{V}_1 = \frac{2\sqrt{3}}{3}, \mathcal{Z}_1 = 0 \tag{31}$$

By substituting the values of the obtained parameters Equation 31 in Equation 11, the following wave solution is obtained:

$$\mathcal{W}(\zeta) = c - \frac{2\sqrt{3}\coth(\zeta)}{3} \tag{32}$$

The graphical diagram related to the solution of the wave $\mathcal{W}(\zeta)$ while $c = -1$ is presented in Figure 9.

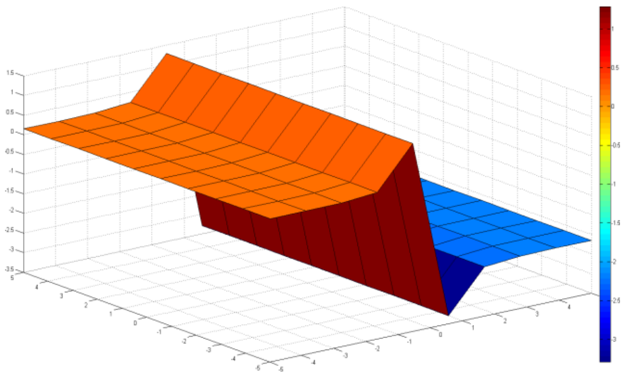

Figure 9. **The graphical diagram related to the solution of Anthology 3-1 with c=-1**

Anthology 3-2:

$$\alpha = -\frac{c^2}{2} + \frac{8}{3}, \beta = 8c - \frac{3}{2}c^3, \mathcal{V}_0 = c,$$
$$\mathcal{V}_1 = \frac{2\sqrt{3}}{3}, \mathcal{Z}_1 = \frac{2\sqrt{3}}{3} \tag{33}$$

By substituting the values of the obtained parameters Equation 33 in Equation 11, the following wave solution is obtained:

$$\mathcal{W} = c - \frac{2\sqrt{3}\coth(\zeta)}{3} \tag{34}$$

The graphical diagram related to the solution of the wave $\mathcal{W}(\zeta)$ while $c = 3$ is presented in Figure 10.

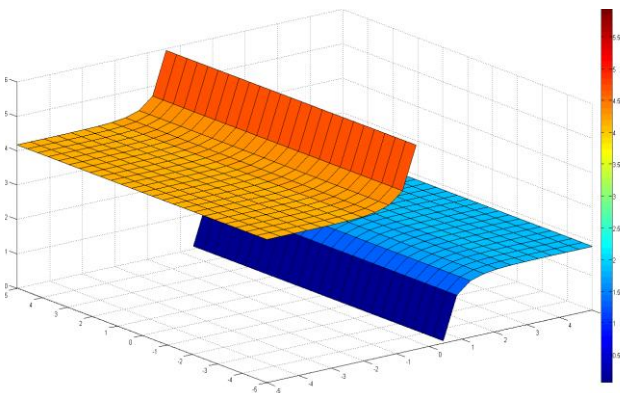

Figure 10. **The graphical diagram related to the solution of Anthology 3-2 with c=3**

Anthology 3-3:

$$\alpha = -\frac{c^2}{2} - \frac{4}{3}, \beta = -4c - \frac{3}{2}c^3, \mathcal{V}_0 = c,$$
$$\mathcal{V}_1 = -\frac{2\sqrt{3}}{3}, \mathcal{Z}_1 = \frac{2\sqrt{3}}{3} \tag{35}$$

By substituting the values of the obtained parameters Equation 35 in Equation 11, the following wave solution is obtained:

$$\mathcal{W}(\zeta) = \frac{2\sqrt{3}\coth(\zeta)^2 + 3c\coth(\zeta) + 2\sqrt{3}}{3\coth(\zeta)} \tag{36}$$

The graphical diagram related to the solution of the wave $\mathcal{W}(\zeta)$ while $c = 5$ is presented in Figure 11.

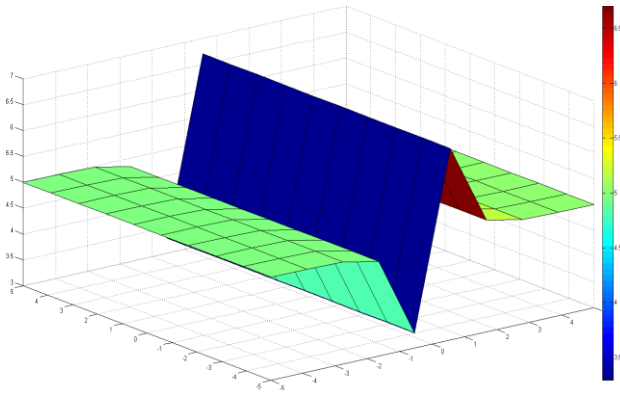

Figure 11. **The graphical diagram related to the solution of Anthology 3-3 with c=5**

Anthology 3-4:

$$\alpha = -\frac{c^2}{2} + \frac{2}{3}, \beta = -\frac{(3c^2-4)\times c}{2}, \mathcal{V}_0 = c,$$
$$\mathcal{V}_1 = 0, \mathcal{Z}_1 = \frac{2\sqrt{3}}{3} \tag{37}$$

By substituting the values of the obtained parameters Equation 37 in Equation 11, the following wave solution is obtained:

$$\mathcal{W}(\zeta) = \frac{3c\coth(\zeta) - 2\sqrt{3}}{3\coth(\zeta)} \tag{38}$$

The graphical diagram related to the solution of the wave $\mathcal{W}(\zeta)$ while $c = 2$ is presented in Figure 12.

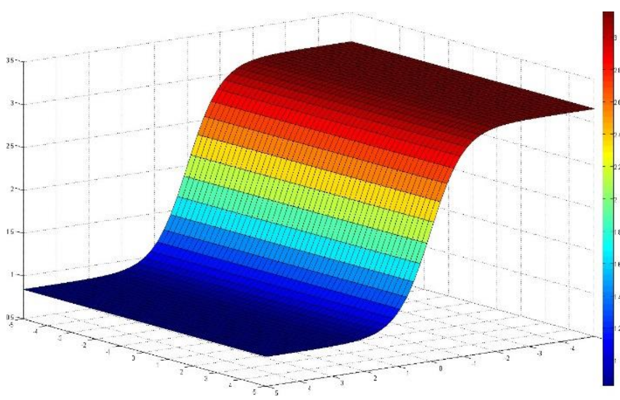

Figure 12. **The graphical diagram related to the solution of Anthology 3-4 with c=5**

Clique 4: catching $[m_1 \quad m_2 \quad m_3 \quad m_4] = [-1 \quad 0 \quad 1 \quad 1]$ and $[\gamma_1 \quad \gamma_1 \quad \gamma_1 \quad \gamma_1] = [0 \quad 0 \quad 0 \quad 1]$ in Equation 4 render:

$$\mathbb{L}(\zeta) = -\frac{1}{1+e^{\zeta}} \tag{39}$$

Anthology 4-1:

$$\alpha = -\frac{c^2}{2} - \frac{1}{3}, \beta = -\frac{(3c^2+2)\times c}{2}, \mathcal{V}_0 = c, \quad \mathcal{V}_1 = 1, \mathcal{Z}_1 = -1 \tag{40}$$

By substituting the values of the obtained parameters Equation 40 in Equation 11, the following wave solution is obtained:

$$\mathcal{W}(\zeta) = \frac{e^{2\zeta}+(c+2)e^{\zeta}+c}{1+e^{\zeta}} \tag{41}$$

The graphical diagram related to the solution of the wave $\mathcal{W}(\zeta)$ while $c = 3$ is presented in Figure 13.

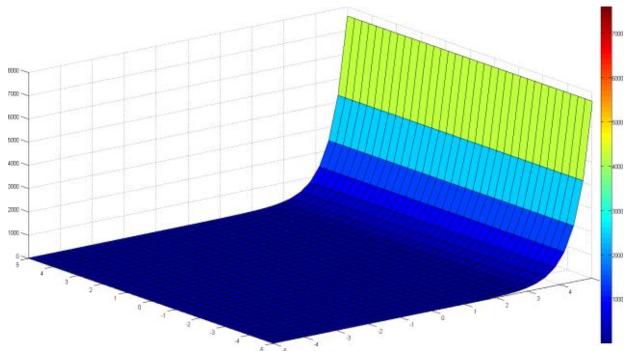

Figure 13. The graphical diagram related to the solution of Anthology 4-1 with c=3

Anthology 4-2:

$$\alpha = \alpha, \beta = -3c^2\mathcal{V}_0 + \frac{9}{2}c\mathcal{V}_0^2 - \frac{3}{2}c\mathcal{V}_0^3 + 3\alpha\frac{9}{2}c\mathcal{V}_0^2, \quad \mathcal{V}_0 = c, \mathcal{V}_1 = 1, \mathcal{Z}_1 = 0 \tag{42}$$

By substituting the values of the obtained parameters Equation 42 in Equation 11, the following wave solution is obtained:

$$\mathcal{W}(\zeta) = \frac{ce^{\zeta}+1}{1+e^{\zeta}} \tag{43}$$

The graphical diagram related to the solution of the wave $\mathcal{W}(\zeta)$ while $c = 1$ is presented in Figure 14.

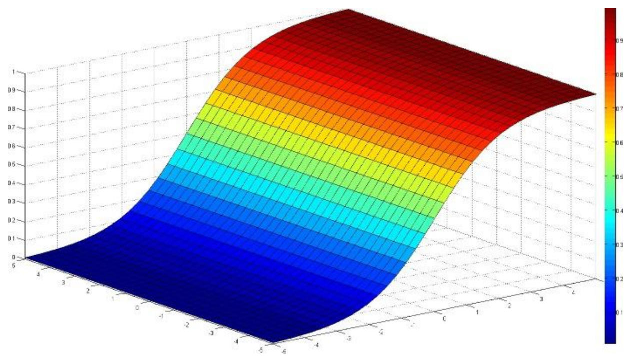

Figure 14. The graphical diagram related to the solution of Anthology 4-2 with c=1

Anthology 4-3:

$$\alpha = -\frac{c^2}{2} - \frac{-1}{3}, \beta = -\frac{(3c^2+2)\times c}{2}, \mathcal{V}_0 = c, \quad \mathcal{V}_1 = \sqrt{-2e^{\zeta} - \frac{2}{3}}, \mathcal{Z}_1 = 0 \tag{44}$$

By substituting the values of the obtained parameters Equation 44 in Equation 11, the following wave solution is obtained:

$$\mathcal{W}(\zeta) = \frac{3ce^{\zeta}+33c-\sqrt{-18e^{\zeta}-6}}{3+3e^{\zeta}} \tag{45}$$

The graphical diagram related to the solution of the wave $\mathcal{W}(\zeta)$ while $c = 2$ is presented in Figure 15.

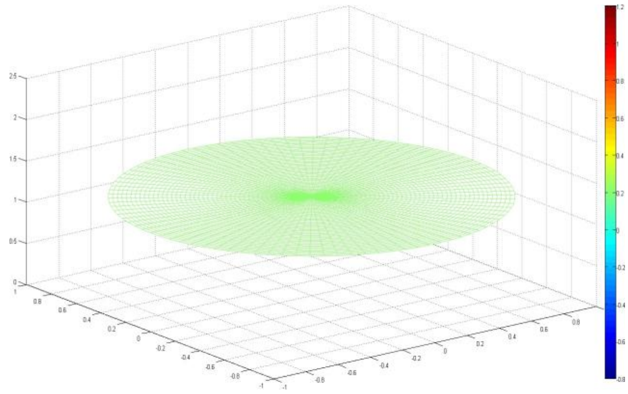

Figure 15. The graphical diagram related to the solution of Anthology 4-3 with c=2

Anthology 4-4:

$$\alpha = -\frac{c^2}{2} - \frac{1}{3}, \beta = -\frac{(3c^2+2)\times c}{2}, \mathcal{V}_0 = c, \quad \mathcal{V}_1 = 0, \mathcal{Z}_1 = -1 \tag{46}$$

By substituting the values of the obtained parameters Equation 46 in Equation 11, the following wave solution is obtained:

$$\mathcal{W}(\zeta) = c + 1 + e^{\zeta} \tag{47}$$

The graphical diagram related to the solution of the wave $\mathcal{W}(\zeta)$ while $c = 0$ is presented in Figure 16.

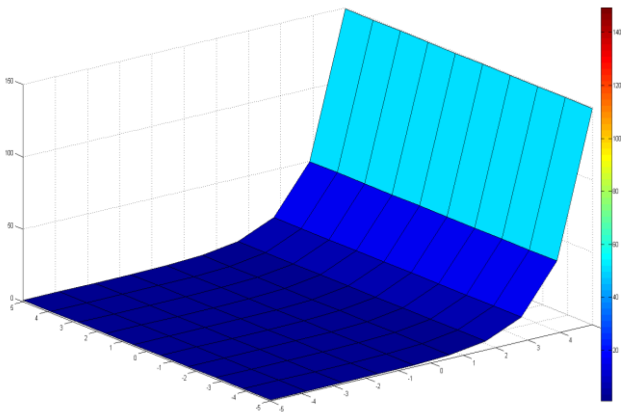

Figure 16. The graphical diagram related to the solution of Anthology 4-4 with c=0

4. Conclusion

At the end of this paper, we were able to reach several exact solutions for the Wu- zhang system using the generalized exponential function method. It can certainly be said that the answers are new, logical, and practical, which promises special progress in the basic sciences and will be a great help to these sciences by using this research as the method is new and there is no special complexity to reach the answer.

5. References

[1] Shah, R., Khan, H., Arif, M., & Kumam, P. (2019). Application of Laplace-Adomian decomposition method for the analytical solution of third-order dispersive fractional partial differential equations. Entropy, 21(4), 335. doi:10.3390/e21040335.

[2] Yang, X. J., & Tenreiro Machado, J. A. (2019). A new fractal nonlinear Burgers' equation arising in the acoustic signals propagation. Mathematical Methods in the Applied Sciences, 42(18), 7539–7544. doi:10.1002/mma.5904.

[3] Touchent, K. A., Hammouch, Z., & Mekkaoui, T. (2020). A modified invariant subspace method for solving partial differential equations with non-singular kernel fractional derivatives. Applied Mathematics and Nonlinear Sciences, 5(2), 35–48. doi:10.2478/amns.2020.2.00012.

[4] Al-Smadi, M., Abu Arqub, O., & Hadid, S. (2020). An attractive analytical technique for a coupled system of fractional partial differential equations in shallow water waves with conformable derivative. Communications in Theoretical Physics, 72(8), 85001. doi:10.1088/1572-9494/ab8a29.

[5] Guo, Y., Cao, X., Liu, B., & Gao, M. (2020). Solving partial differential equations using deep learning and physical constraints. Applied Sciences (Switzerland), 10(17), 5917. doi:10.3390/app10175917.

[6] Siddique, I., Jaradat, M. M. M., Zafar, A., Bukht Mehdi, K., & Osman, M. S. (2021). Exact traveling wave solutions for two prolific conformable M-Fractional differential equations via three diverse approaches. Results in Physics, 28, 104557. doi:10.1016/j.rinp.2021.104557.

[7] Schiassi, E., Furfaro, R., Leake, C., De Florio, M., Johnston, H., & Mortari, D. (2021). Extreme theory of functional connections: A fast physics-informed neural network method for solving ordinary and partial differential equations. Neurocomputing, 457, 334–356. doi:10.1016/j.neucom.2021.06.015.

[8] Talib, I., Jarad, F., Mirza, M. U., Nawaz, A., & Riaz, M. B. (2022). A generalized operational matrix of mixed partial derivative terms with applications to multi-order fractional partial differential equations. Alexandria Engineering Journal, 61(1), 135–145. doi:10.1016/j.aej.2021.04.067.

[9] Özkan, E. M., & Özkan, A. (2023). On exact solutions of some important nonlinear conformable time-fractional differential equations. SeMA Journal, 80(2), 303–318. doi:10.1007/s40324-022-00290-5.

[10] Fibich, G. (2015). The Nonlinear Schrödinger Equation. In Applied Mathematical Sciences. Springer International Publishing, Cham, Switzerland. doi:10.1007/978-3-319-12748-4.

[11] Rizvi, S. T. R., Ali, K., & Ahmad, M. (2020). Optical solitons for Biswas–Milovic equation by new extended auxiliary equation method. Optik, 204. doi:10.1016/j.ijleo.2020.164181.

[12] Veeresha, P., Prakasha, D. G., Singh, J., Kumar, D., & Baleanu, D. (2020). Fractional Klein-Gordon-Schrödinger equations with Mittag-Leffler memory. Chinese Journal of Physics, 68, 65–78. doi:10.1016/j.cjph.2020.08.023.

[13] Düşünceli, F. (2018). Solutions for the Drinfeld-Sokolov equation using an IBSEFM method. Muş Alparslan Üniversitesi Fen Bilimleri Dergisi, 6(1), 505-510.

[14] Adler, V. E. (2011). On a discrete analog of the Tzitzeica equation. arXiv preprint arXiv:1103.5139. doi:10.48550/arXiv.1103.5139.

[15] Triki, H., Hayat, T., & Aldossary, O. M. (2012). 1-Soliton Solution of the Three Component System of Wu-Zhang Equations ABSTRACT| FULL TEXT. Hacettepe Journal of Mathematics and Statistics, 41(4), 537-543.

[16] Chen, C.-L., Tang, X., & Lou, S.-Y. (2002). Solutions of a (2+1)-dimensional dispersive long wave equation. Physical Review E, 66(3). doi:10.1103/physreve.66.036605.

[17] Zheng, X., Chen, Y., & Zhang, H. (2003). Generalized extended tanh-function method and its application to (1+1)-dimensional dispersive long wave equation. Physics Letters A, 311(2–3), 145–157. doi:10.1016/s0375-9601(03)00451-1.

[18] Taghizadeh, N., Akbari, M., & Shahidi, M. (2011). Application of reduced differential transform method to the Wu-Zhang equation. Australian journal of basic and applied sciences, 5(5), 565-571.

[19] Eslami, M., & Rezazadeh, H. (2016). The first integral method for Wu–Zhang system with conformable time-fractional derivative. Calcolo, 53(3), 475–485. doi:10.1007/s10092-015-0158-8.

[20] Inc, M., Kilic, B., Karatas, E., Mohamed Al Qurashi, M., Baleanu, D., & Tchier, F. (2016). On soliton solutions of the Wu-Zhang system. Open Physics, 14(1), 76–80. doi:10.1515/phys-2016-0004.

[21] Xiong, N., Li, Y. Q., Chen, J. C., & Chen, Y. (2016). One-Dimensional Optimal System and Similarity Reductions of Wu - Zhang Equation. Communications in Theoretical Physics, 66(1), 1–11. doi:10.1088/0253-6102/66/1/001.

[22] Mirzazadeh, M., Ekici, M., Eslamic, M., Krishnan, E. V., Kumar, S., & Biswas, A. (2017). Solitons and other solutions to Wu–Zhang system. Nonlinear Analysis: Modelling and Control, 22(4), 441–458. doi:10.15388/NA.2017.4.2.

[23] Koparan, M., Kaplan, M., Bekir, A., & Guner, O. (2017). A novel generalized Kudryashov method for exact solutions of nonlinear evolution equations. AIP Conference Proceedings, 1798(1)). doi:10.1063/1.4972674.

[24] Kaplan, M., Mayeli, P., & Hosseini, K. (2017). Exact traveling wave solutions of the Wu–Zhang system

describing (1 + 1)-dimensional dispersive long wave. Optical and Quantum Electronics, 49(12), 1–10. doi:10.1007/s11082-017-1231-0.

[25] Awan, A. U., Tahir, M., & Rehman, H. U. (2019). On traveling wave solutions: The Wu-Zhang system describing dispersive long waves. Modern Physics Letters B, 33(6), 1950059. doi:10.1142/S0217984919500593.

[26] Yel, G., & Baskonus, H. M. (2019). Solitons in conformable time-fractional Wu–Zhang system arising in coastal design. Pramana - Journal of Physics, 93(4). doi:10.1007/s12043-019-1818-z.

[27] Ghanbari, B., & Inc, M. (2018). A new generalized exponential rational function method to find exact special solutions for the resonance nonlinear Schrödinger equation. European Physical Journal Plus, 133(4), 142. doi:10.1140/epjp/i2018-11984-1.

[28] Ghanbari, B., & Gómez-Aguilar, J. F. (2019). The generalized exponential rational function method for Radhakrishnan-Kundu-Lakshmanan equation with β-conformable time derivative. Revista Mexicana de Fisica, 65(5), 503–518. doi:10.31349/RevMexFis.65.503.

[29] Ghanbari, B. (2019). Abundant soliton solutions for the Hirota-Maccari equation via the generalized exponential rational function method. Modern Physics Letters B, 33(9), 1950106. doi:10.1142/S0217984919501069.

[30] Ghanbari, B., & Nisar, K. S. (2020). Determining new soliton solutions for a generalized nonlinear evolution equation using an effective analytical method. Alexandria Engineering Journal, 59(5), 3171–3179. doi:10.1016/j.aej.2020.07.032.

[31] Ghanbari, B., & Kuo, C. K. (2020). A variety of solitary wave solutions to the (2+1)-dimensional bidirectional SK and variable-coefficient SK equations. Results in Physics, 18, 103266. doi:10.1016/j.rinp.2020.103266.

[32] Cao, Y., Parvaneh, F., Alamri, S., Rajhi, A. A., & Anqi, A. E. (2021). Some exact wave solutions to a variety of the Schrödinger equation with two nonlinearity laws and conformable derivative. Results in Physics, 31, 104929. doi:10.1016/j.rinp.2021.104929.

[33] Günay, B., Kuo, C. K., & Ma, W. X. (2021). An application of the exponential rational function method to exact solutions to the Drinfeld–Sokolov system. Results in Physics, 29, 104733. doi:10.1016/j.rinp.2021.104733.

[34] Ali, K. K., Yilmazer, R., Bulut, H., Aktürk, T., & Osman, M. S. (2021). Abundant exact solutions to the strain wave equation in micro-structured solids. Modern Physics Letters B, 35(26), 2150439. doi:10.1142/s021798492150439x.