Application of Generalized Exponential Function Method for Exact Solutions of Wu-Zhang System

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Abstract:

The present research aims to find a precise method for the wu-zhang system on scattered long waves, which can be a positive step for physical science in dealing with the structure of scattered waves and provide 3D diagrams for further studies of other sciences.

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1. Introduction

One of the most popular, widely used, and diverse mathematical equations is differential equations with partial derivatives, which sometimes attract the attention of many researchers and mathematical scientists who, by researching these equations and completing the research of others, arrive at remarkable methods and techniques that are a significant help to other sciences significantly in finding the answers to these equations.

Among the methods used for differential equations with partial derivatives that have been obtained over many years and as a result of the experiences of many scientists, Rasul Shah et al. 2019, used the Laplace-Adomin decomposition method to get the solutions of partial differential equations. They investigated the third-order diffuse fraction [1] and also applied the traveling wave transform of the non-differentiable type in 2019. Yang and Tenreiro Machado [2] discussed the exact solutions to the new nonlinear Burgers equation. In 2020, Touchent et al. applied a modified fixed subspace method to obtain the solution of partial differential equations with fractional derivatives of non-singular kernels [3], an attractive method for coupled systems of fractional partial differential equations which was proposed in 2020 by Al-Smadi et al.[4]. Guo et al. (2020) conducted research

on deep learning and physical constraints for solving partial differential equations [5]. In 2021, three common methods of two M-fractional differential equations were adopted by Siddique et al., who applied and compared them [6]. A new neural network method for solving ordinary and partial differential equations was used in 2021 by Schiassi et al. [7], a generalized operational matrix to obtain the solutions of differential equations. The fractional fraction was investigated several times in 2022 by Imran Talib et al. [8]. In 2023, Ozkan and Ozkan presented a discussion of the exact solutions of a type of differential equation [9] and many pieces of research that have been carried out in previous years or later by many researchers.

Various forms of nonlinear partial differential equations have been studied for this purpose. For example, the nonlinear Schrö¨dinger equation [10], the Biswas- Milock equation [11], the Schrö¨ dinger-Klein-Gordon (SKG) equation [12], the Dihrenfeld-Sokoloff equations [13], Tzitzika-type evolutionary equations [14], and many others.

This research investigates and studies one of these PDEs that appears in the dynamics of long scattered waves, known as the Wu Zhang model [15]. To solve this system, various powerful techniques and methods have been used:

- In 2002, Chen et al., with the help of Jacobi elliptic functions [16],

- In 2003, Zheng et al. and with the Extended tanh-function method [17],

- In 2011, Taghizadeh et al. and with the Reduced differential transform method [18],

- In 2015, Eslami et al., with the First integral method [19],

- In 2016, Inc et al. used two extended Tanh methods: the Hirota method [20]

- In 2016, Xiong et al., using the Lie symmetry method [21],

- In 2017, Mirzazadeh et al., by a method based on Lie symmetry [22],

- In 2017, Koparan et al. with the Generalized Kudryashov Method [23],

- In 2017, Kaplan et al., with the Exponential rational function method [24],

- In 2019, by Awan et al., using the generalized (G0=G) expansion method [25],

- In 2019, Yel and Baskonus, his colleague, modified the exp-expansion function method [26] and …

In the current research, we use the generalized exponential function method that Ghanbari and Inc. studied in 2018 [27] to reach new and accurate solutions for the wu-zhang system. This generalized exponential function method has already been used by Ghanbari and several researchers for many equations [28-34]. The outline of this work is as follows: In the second part, the key concepts of the generalized exponential function method are introduced. In the third part, we use this method for the above system. The fourth section includes results and discussion.

2. Demarche

The method used in the present research is a method that has been widely used in solving many PDE schemes, which we now use to obtain the solutions of the Wu-Zhang system model. To illustrate the technique used in this research, let us presume a framework as conform below:

$$H(X, X_x, X_t, X_{xx}, \cdots) = 0 \tag{1}$$

Using the wave $X(t, h) = X(\zeta)$, $\zeta = t - kh$, Equation 1 becomes a NODE equation

$$H\left(X(\zeta), \frac{dX}{dX'}, \frac{d^2X}{dX^{2'}}, \cdots\right) = 0 \tag{2}$$

This technique is built using a hypothetical solution that can be displayed as follows:

$$\mathcal{W}(\zeta) = \mathcal{V}_0 + \sum_{i=0}^{a_0} \mathcal{V}_i \mathbb{L}^i(\zeta) + \sum_{i=0}^{a_0} \frac{z_i}{\mathbb{L}^i(\zeta)} \tag{3}$$

where

$$\zeta = \frac{m_1 \exp(\gamma_1 \zeta) + m_2 \exp(\gamma_2 \zeta)}{m_3 \exp(\gamma_3 \zeta) + m_4 \exp(\gamma_4 \zeta)} \tag{4}$$

In Equation 4, $\mathcal{V}_0$, $\mathcal{V}_i$, Z_i $(1 \leq i \leq a_0)$, and in addition $m_i, \gamma_i, (1 \leq i \leq 4)$, to determine the positive integer a_0, we can use some well-known balance rules. By inserting Equation 3 in Equation 2, we get a polynomial. Finally, analytical solutions for Equation 1 are obtained.

3. Solution of Equation 1 with Demarche Suggested in Section 2

The Wu Zhang equation (WZ) studied in the present study is as follows:

$$\begin{cases} u_t + u_{xx} + \mathcal{W}u_y + X_x = 0 \\ \mathcal{W}_t + u\mathcal{W}_x + \mathcal{W}_y + X_y = 0 \\ X_t + (uX)_x + (\mathcal{W}X)_y + \frac{1}{3}(u_{xxx} + u_{xxy} + \mathcal{W}_{xxy} + \mathcal{W}_{yyy}) = 0 \end{cases} \tag{5}$$

By changing the scale and reducing symmetry, Equation 5 can be reduced to Equation 6:

$$\begin{cases} \mathcal{W}_t + \mathcal{W}\mathcal{W}_x + X_x = 0 \\ X_t + X\mathcal{W}_x + \mathcal{W}X_x + \frac{1}{3}\mathcal{W}_{xxx} = 0 \end{cases} \tag{6}$$

In this section, we want to find the exact solutions of the equation (WZ). For this purpose, from the traveling wave transformation $\mathcal{W}(t, h) = \mathcal{W}(\zeta)$, $X(t, h) = X(\zeta)$, $\zeta = t - kh$, system 6 is reduced to the following ODE system:

$$-c\mathcal{W}' + \mathcal{W}\mathcal{W}' + X' = 0 \tag{7}$$

$$-cX' + X\mathcal{W}' + \mathcal{W}X' + \frac{1}{3}\mathcal{W}''' = 0 \tag{8}$$

By integrating Equation 8 once and substituting X into Equation 9, we find:

$$(-3c^2 + 3\alpha)\mathcal{W}' + 9c\mathcal{W}\mathcal{W}' - \frac{9}{2}\mathcal{W}\mathcal{W}' + \mathcal{W}''' = 0 \tag{9}$$

$$\mathcal{W}'' - \beta - (3c^2 - 3\alpha)\mathcal{W} - \frac{9}{2}c\mathcal{W}^2 + \frac{3}{2}\mathcal{W}^3 = 0 \tag{10}$$

According to the balance principle, the value of a_0 is 1, so:

$$\mathcal{W}(\zeta) = \mathcal{V}_0 + \mathcal{V}_1 \mathbb{L}(\zeta) + \frac{Z_1}{\mathbb{L}(\zeta)} \tag{11}$$

That $\mathbb{L}(\zeta)$ is obtained according to Equation 4.

Clique 1: catching $[m_1 \ m_2 \ m_3 \ m_4] = [-1 \ 1 \ 1 \ 1]$ and $[\gamma_1 \ \gamma_1 \ \gamma_1 \ \gamma_1] = [1 \ -1 \ 1 \ -1]$ in Equation 4 render:

$$\mathbb{L}(\zeta) = -tanh\zeta \tag{12}$$

Various anthologies are obtained as follows:

Anthology 1-1:

$$\alpha = -\frac{c^2}{2} + \frac{2}{3}, \beta = -\frac{(3c^2) \times c}{2}, \mathcal{V}_0 = c,$$
$$\mathcal{V}_1 = \frac{2\sqrt{3}}{3}, Z_1 = 0 \tag{13}$$

By substituting the values of the obtained parameters Equation 13 in Equation 11, the following wave solution is obtained:

$$\mathcal{W} = c - \frac{2\sqrt{3}\tanh(\zeta)}{3} \tag{14}$$

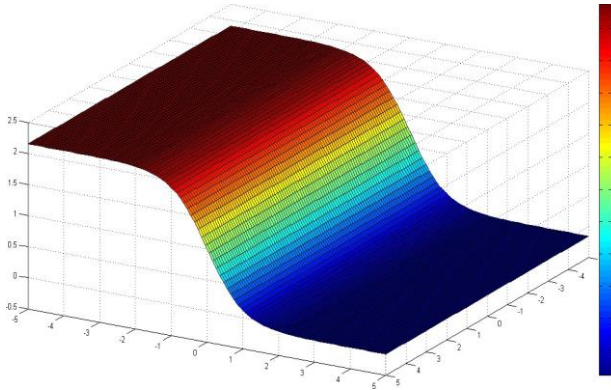The graphical diagram related to the solution of the wave $\mathcal{W}(\zeta)$ while $c = 1$ is presented in Figure 1.

Figure 1. The graphical diagram related to the solution of Anthology 1-1 with c=1

Anthology 1-2:

$$\alpha = -\frac{c^2}{2} + \frac{8}{3}, \beta = -\frac{(3c^2+16)\times c}{2}, \mathcal{V}_0 = c,$$
$$\mathcal{V}_1 = \frac{2\sqrt{3}}{3}, \mathcal{Z}_1 = \frac{2\sqrt{3}}{3} \tag{15}$$

By substituting the values of the obtained parameters Equation 15 in Equation 11, the following wave solution is obtained:

$$\mathcal{W}(\zeta) = \frac{2\sqrt{3}\tanh(\zeta)^2 + 3c\tanh(\zeta) - 2\sqrt{3}}{3\tanh(\zeta)} \tag{16}$$

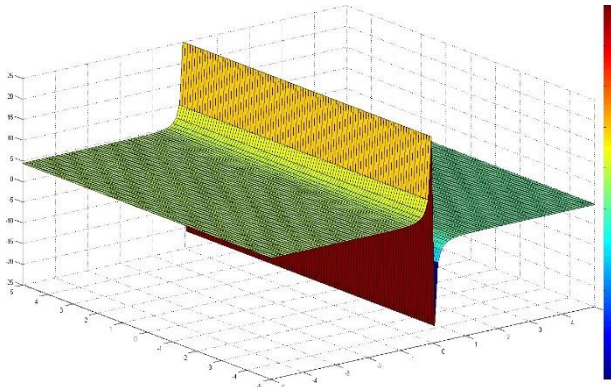The graphical diagram related to the solution of the wave $\mathcal{W}(\zeta)$ while $c = 2$ is presented in Figure 2.

Figure 2. The graphical diagram related to the solution of Anthology 1-2 with c=2

Anthology 1-3:

$$\alpha = -\frac{c^2}{2} - \frac{4}{3}, \beta = -4c\frac{3}{2}c^3, \mathcal{V}_0 = c,$$
$$\mathcal{V}_1 = \frac{2\sqrt{3}}{3}, \mathcal{Z}_1 = -\frac{2\sqrt{3}}{3} \tag{17}$$

By substituting the values of the obtained parameters Equation 17 in Equation 11, the following wave solution is obtained:

$$\mathcal{W}(\zeta) = \frac{-2\sqrt{3}\tanh(\zeta)^2 + 3c\tanh(\zeta) + 2\sqrt{3}}{3\tanh(\zeta)} \tag{18}$$

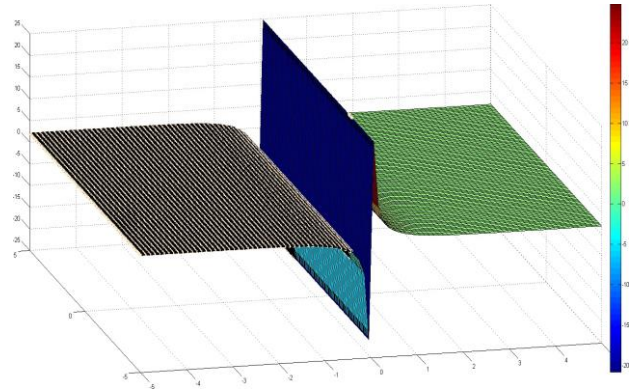The graphical diagram related to the solution of the wave $\mathcal{W}(\zeta)$ while $c = 2$ is presented in Figure 3.

Figure 3. The graphical diagram related to the solution of Anthology 1-3 with c=2

Anthology 1-4:

$$\alpha = -\frac{c^2}{2} + \frac{2}{3}, \beta = -\frac{(3c^2-4)\times c}{2}, \mathcal{V}_0 = c,$$
$$\mathcal{V}_1 = 0, \mathcal{Z}_1 = -\frac{2\sqrt{3}}{3} \tag{19}$$

By substituting the values of the obtained parameters Equation 19 in Equation 11, the following wave solution is obtained:

$$\mathcal{W}(\zeta) = \frac{3\tanh(\zeta) - 2\sqrt{3}}{3\tanh(\zeta)} \tag{20}$$

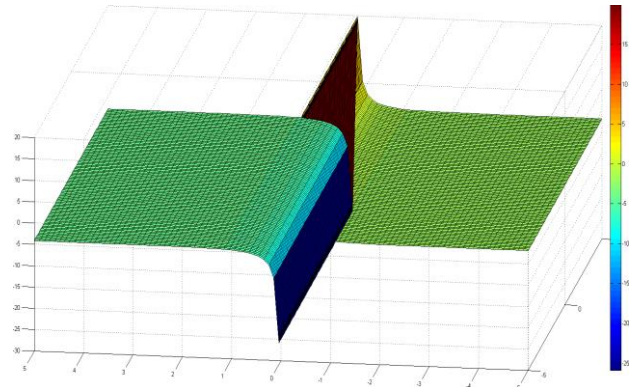The graphical diagram related to the solution of the wave $\mathcal{W}(\zeta)$ while $c = 3$ is presented in Figure 4.

Figure 4. The graphical diagram related to the solution of Anthology 1-4 with c=3

Clique 2: catching $[m_1 \quad m_2 \quad m_3 \quad m_4] = [i \quad -i \quad 1 \quad 1]$ and $[\gamma_1 \quad \gamma_1 \quad \gamma_1 \quad \gamma_1] = [i \quad -i \quad i \quad -i]$ in Equation 4 render:

$$\mathbb{L}(\zeta) = -tanh\zeta \tag{21}$$

Anthology 2-1:

$$\alpha = -\frac{c^2}{2} - \frac{2}{3}, \beta = -\frac{(3c^2-4)\times c}{2}, \mathcal{V}_0 = c, \tag{22}$$

$$\mathcal{V}_1 = -\frac{2\sqrt{3}}{3}, \mathcal{Z}_1 = 0$$

By substituting the values of the obtained parameters Equation 22 in Equation 11, the following wave solution is obtained:

$$\mathcal{W} = c - \frac{2\sqrt{3}\tanh(\zeta)}{3} \tag{23}$$

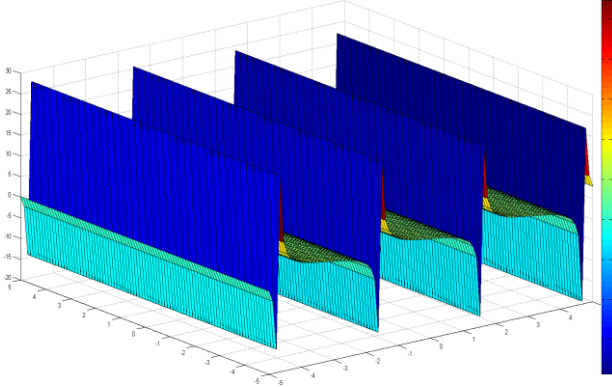The graphical diagram related to the solution of the wave $\mathcal{W}(\zeta)$ while $c = 4$ is presented in Figure 5.

Figure 5. The graphical diagram related to the solution of Anthology 2-1 with c=4

Anthology 2-2:

$$\alpha = -\frac{c^2}{2} - \frac{8}{3}, \beta = -8c - \frac{3}{2}c^3, \mathcal{V}_0 = c,$$
$$\mathcal{V}_1 = -\frac{2\sqrt{3}}{3}, \mathcal{Z}_1 = \frac{2\sqrt{3}}{3} \tag{24}$$

By substituting the values of the obtained parameters Equation 24 in Equation 11, the following wave solution is obtained:

$$\mathcal{W}(\zeta) = \frac{2\sqrt{3}\tanh(\zeta)^2 + 3c\tanh(\zeta) - 2\sqrt{3}}{3\tanh(\zeta)} \tag{25}$$

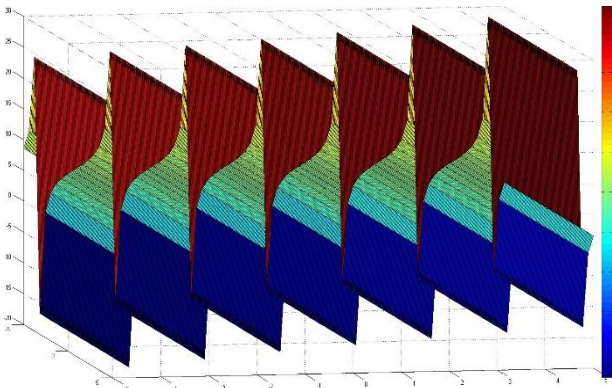The graphical diagram related to the solution of the wave $\mathcal{W}(\zeta)$ while $c = 5$ is presented in Figure 6.

Figure 6. The graphical diagram related to the solution of Anthology 2-2 with c=5

Anthology 2-3:

$$\alpha = -\frac{c^2}{2} + \frac{4}{3}, \beta = 4c - \frac{3}{2}c^3, \mathcal{V}_0 = c, \tag{26}$$

$$\mathcal{V}_1 = -\frac{2\sqrt{3}}{3}, \mathcal{Z}_1 = -\frac{2\sqrt{3}}{3}$$

By substituting the values of the obtained parameters Equation 26 in Equation 11, the following wave solution is obtained:

$$\mathcal{W} = \frac{2\sqrt{3}\tanh(\zeta)^2 + 3c\tanh(\zeta) + 2\sqrt{3}}{3\tanh(\zeta)} \tag{27}$$

The graphical diagram related to the solution of the wave $\mathcal{W}(\zeta)$ while $c = 5$ is presented in Figure 7.
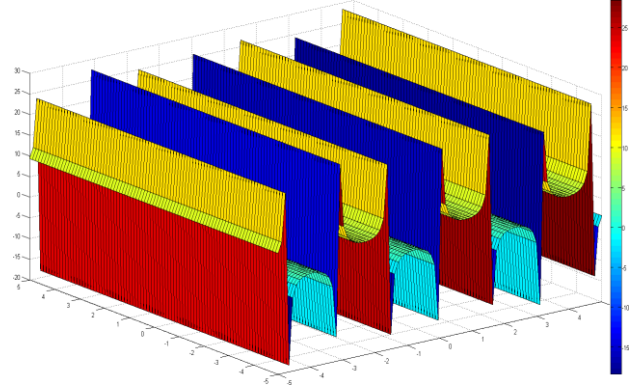

Figure 7. The graphical diagram related to the solution of Anthology 2-3 with c=5

Anthology 2-4:

$$\alpha = -\frac{c^2}{2} - \frac{2}{3}, \beta = -\frac{(3c^2 + 4) \times c}{2}, \mathcal{V}_0 = c,$$
$$\mathcal{V}_1 = 0, \mathcal{Z}_1 = -\frac{2\sqrt{3}}{3} \tag{28}$$

By substituting the values of the obtained parameters Equation 28 in Equation 11, the following wave solution is obtained:

$$\mathcal{W}(\zeta) = \frac{3\tanh(\zeta) + 2\sqrt{3}}{3\tanh(\zeta)} \tag{29}$$

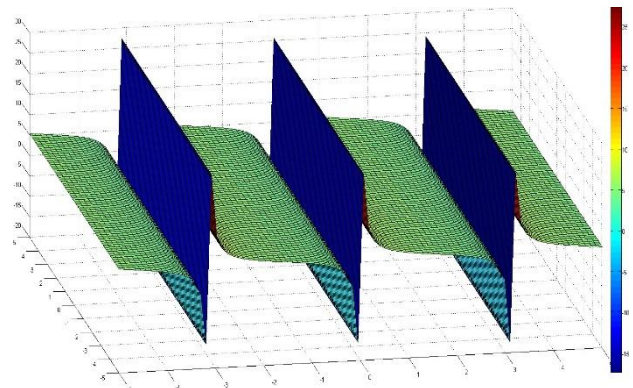The graphical diagram related to the solution of the wave $\mathcal{W}(\zeta)$ while $c = 3$ is presented in Figure 8.

Figure 8. The graphical diagram related to the solution of Anthology 2-4 with c=1

Clique 3: catching $[m_1 \quad m_2 \quad m_3 \quad m_4] = [1 \quad 1 \quad -1 \quad 1]$ and $[\gamma_1 \quad \gamma_1 \quad \gamma_1 \quad \gamma_1] = [1 \quad -1 \quad 1 \quad -1]$ in Equation 4 render:

$$\mathbb{L}(\zeta) = -coth\zeta \tag{30}$$

Anthology 3-1:

$$\alpha = -\frac{c^2}{2} + \frac{2}{3}, \beta = -\frac{(3c^2-4)\times c}{2}, \mathcal{V}_0 = c,$$
$$\mathcal{V}_1 = \frac{2\sqrt{3}}{3}, \mathcal{Z}_1 = 0 \tag{31}$$

By substituting the values of the obtained parameters Equation 31 in Equation 11, the following wave solution is obtained:

$$\mathcal{W}(\zeta) = c - \frac{2\sqrt{3}\coth(\zeta)}{3} \tag{32}$$

The graphical diagram related to the solution of the wave $\mathcal{W}(\zeta)$ while $c = -1$ is presented in Figure 9.
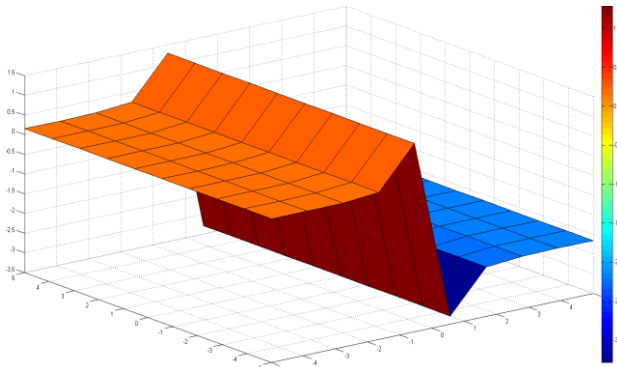

Figure 9. The graphical diagram related to the solution of Anthology 3-1 with c=-1

Anthology 3-2:

$$\alpha = -\frac{c^2}{2} + \frac{8}{3}, \beta = 8c - \frac{3}{2}c^3, \mathcal{V}_0 = c,$$
$$\mathcal{V}_1 = \frac{2\sqrt{3}}{3}, \mathcal{Z}_1 = \frac{2\sqrt{3}}{3} \tag{33}$$

By substituting the values of the obtained parameters Equation 33 in Equation 11, the following wave solution is obtained:

$$\mathcal{W} = c - \frac{2\sqrt{3}\coth(\zeta)}{3} \tag{34}$$

The graphical diagram related to the solution of the wave $\mathcal{W}(\zeta)$ while $c = 3$ is presented in Figure 10.

Figure 10. The graphical diagram related to the solution of Anthology 3-2 with c=3

Anthology 3-3:

$$\alpha = -\frac{c^2}{2} - \frac{4}{3}, \beta = -4c - \frac{3}{2}c^3, \mathcal{V}_0 = c,$$
$$\mathcal{V}_1 = -\frac{2\sqrt{3}}{3}, \mathcal{Z}_1 = \frac{2\sqrt{3}}{3} \tag{35}$$

By substituting the values of the obtained parameters Equation 35 in Equation 11, the following wave solution is obtained:

$$\mathcal{W}(\zeta) = \frac{2\sqrt{3}\coth(\zeta)^2 + 3c\coth(\zeta) + 2\sqrt{3}}{3\coth(\zeta)} \tag{36}$$

The graphical diagram related to the solution of the wave $\mathcal{W}(\zeta)$ while $c = 5$ is presented in Figure 11.
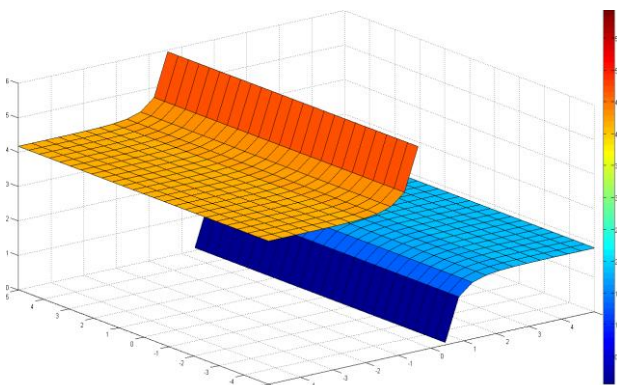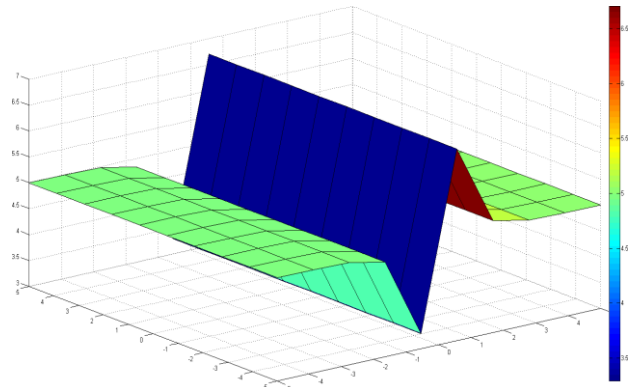

Figure 11. The graphical diagram related to the solution of Anthology 3-3 with c=5

Anthology 3-4:

$$\alpha = -\frac{c^2}{2} + \frac{2}{3}, \beta = -\frac{(3c^2-4)\times c}{2}, \mathcal{V}_0 = c,$$
$$\mathcal{V}_1 = 0, \mathcal{Z}_1 = \frac{2\sqrt{3}}{3} \tag{37}$$

By substituting the values of the obtained parameters Equation 37 in Equation 11, the following wave solution is obtained:

$$\mathcal{W}(\zeta) = \frac{3c\coth(\zeta) - 2\sqrt{3}}{3\coth(\zeta)} \tag{38}$$

The graphical diagram related to the solution of the wave $\mathcal{W}(\zeta)$ while $c = 2$ is presented in Figure 12.
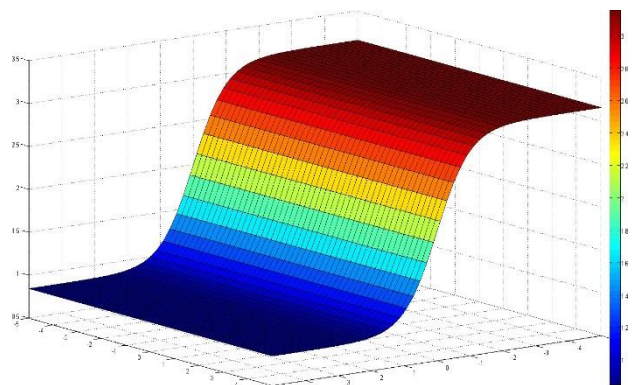

Figure 12. The graphical diagram related to the solution of Anthology 3-4 with c=5

Clique 4: catching $[m_1 \quad m_2 \quad m_3 \quad m_4] = [-1 \quad 0 \quad 1 \quad 1]$ and $[\gamma_1 \quad \gamma_1 \quad \gamma_1 \quad \gamma_1] = [0 \quad 0 \quad 0 \quad 1]$ in Equation 4 render:

$$\mathbb{L}(\zeta) = -\frac{1}{1+e^\zeta} \tag{39}$$

Anthology 4-1:

$$\alpha = -\frac{c^2}{2} - \frac{1}{3}, \beta = -\frac{(3c^2+2)\times c}{2}, \mathcal{V}_0 = c,$$
$$\mathcal{V}_1 = 1, \mathcal{Z}_1 = -1 \tag{40}$$

By substituting the values of the obtained parameters Equation 40 in Equation 11, the following wave solution is obtained:

$$\mathcal{W}(\zeta) = \frac{e^{2\zeta}+(c+2)e^\zeta+c}{1+e^\zeta} \tag{41}$$

The graphical diagram related to the solution of the wave $\mathcal{W}(\zeta)$ while $c = 3$ is presented in Figure 13.
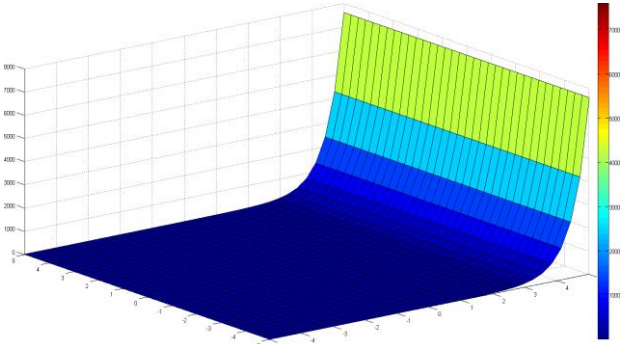

Figure 13. The graphical diagram related to the solution of Anthology 4-1 with c=3

Anthology 4-2:

$$\alpha = \alpha, \beta = -3c^2\mathcal{V}_0 + \frac{9}{2}c\mathcal{V}_0^2 - \frac{3}{2}c\mathcal{V}_0^3 + 3\alpha\frac{9}{2}c\mathcal{V}_0^2,$$
$$\mathcal{V}_0 = c, \mathcal{V}_1 = 1, \mathcal{Z}_1 = 0 \tag{42}$$

By substituting the values of the obtained parameters Equation 42 in Equation 11, the following wave solution is obtained:

$$\mathcal{W}(\zeta) = \frac{ce^\zeta+1}{1+e^\zeta} \tag{43}$$

The graphical diagram related to the solution of the wave $\mathcal{W}(\zeta)$ while $c = 1$ is presented in Figure 14.

Figure 14. The graphical diagram related to the solution of Anthology 4-2 with c=1

Anthology 4-3:

$$\alpha = -\frac{c^2}{2} - \frac{-1}{3}, \beta = -\frac{(3c^2+2)\times c}{2}, \mathcal{V}_0 = c,$$
$$\mathcal{V}_1 = \sqrt{-2e^\zeta - \frac{2}{3}}, \mathcal{Z}_1 = 0 \tag{44}$$

By substituting the values of the obtained parameters Equation 44 in Equation 11, the following wave solution is obtained:

$$\mathcal{W}(\zeta) = \frac{3ce^\zeta+33c-\sqrt{-18e^\zeta-6}}{3+3e^\zeta} \tag{45}$$

The graphical diagram related to the solution of the wave $\mathcal{W}(\zeta)$ while $c = 2$ is presented in Figure 15.
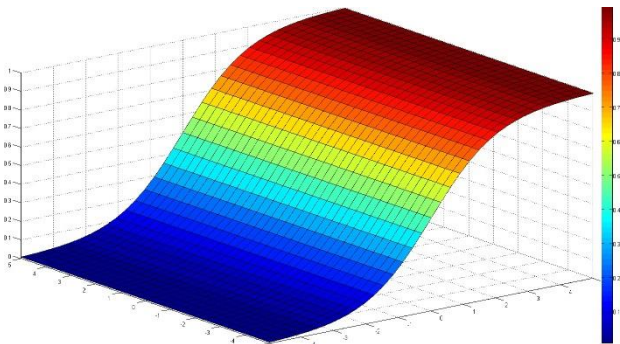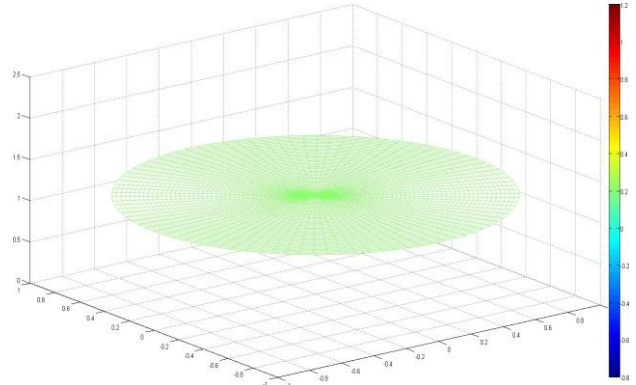

Figure 15. The graphical diagram related to the solution of Anthology 4-3 with c=2

Anthology 4-4:

$$\alpha = -\frac{c^2}{2} - \frac{1}{3}, \beta = -\frac{(3c^2+2)\times c}{2}, \mathcal{V}_0 = c,$$
$$\mathcal{V}_1 = 0, \mathcal{Z}_1 = -1 \tag{46}$$

By substituting the values of the obtained parameters Equation 46 in Equation 11, the following wave solution is obtained:

$$\mathcal{W}(\zeta) = c + 1 + e^\zeta \tag{47}$$

The graphical diagram related to the solution of the wave $\mathcal{W}(\zeta)$ while $c = 0$ is presented in Figure 16.
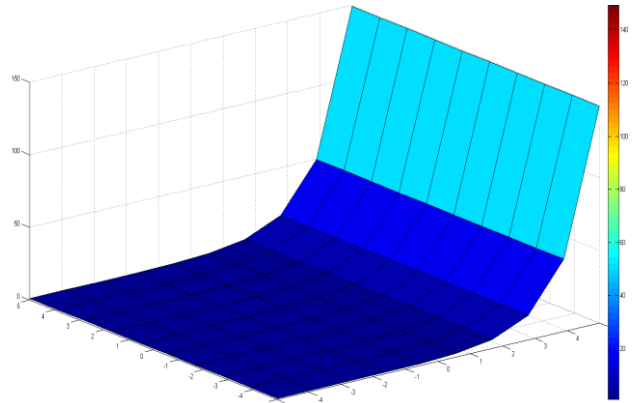

Figure 16. The graphical diagram related to the solution of Anthology 4-4 with c=0

4. Conclusion

At the end of this paper, we were able to reach several exact solutions for the Wu- zhang system using the generalized exponential function method. It can certainly be said that the answers are new, logical, and practical, which promises special progress in the basic sciences and will be a great help to these sciences by using this research as the method is new and there is no special complexity to reach the answer.

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